



## Self exciting threshold auto-regressive approach for non-linear modeling of daily electricity prices in the selected regions

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### Abstract

*This paper is focused on the electricity market and electricity prices. The electricity sector is one of the key strategic sectors of every economy and knowledge of demand, supply and prices is very important. Because of the features occurring in the time series of electricity prices (i.e. high frequency, non-constant mean, autocorrelation, non-normal distribution, heteroscedasticity, seasonality, etc.), it is necessary to employ more sophisticated models for the purposes of their modeling. The goal of this paper is to propose the empirical model for modeling daily electricity prices in three selected regions (California, North Europe and Austria). To exploit non-linearity, we apply the SETAR (Self Exciting Threshold Auto-Regressive) models that imply and distinct regimes in time series dynamics with potentially different parameters (and thus dynamics properties) of each regime. First, the most appropriate SETAR model for modeling electricity prices at selected markets is developed; next, statistical verification of each model is performed in accordance with Hansen (1997, 2000); finally, it is verified whether the proposed non-linear models give satisfactory results in the sense of data fitting and diagnostic checks.*

### Keywords

*Electricity, electricity price, non-linear time series, regime-switching model, SETAR model.*

**JEL Classification:** C1, C13, C52, G1

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### 1. Introduction

In many parts of the world, the sector of electricity generation is gradually converting to the competitive market structure replacing the traditional monopolistic environment and therefore there arises a need to model the time series of electricity price for selected groups of market participants.

An accurate modeling (possibly forecasting) of electricity prices including analyzing the factors affecting these prices has become a very important tool for generators and consumers. In a short time period, a generating company needs to model

electricity prices to set its generating strategy and to optimally schedule energy resources. This is important for profit planning and that is why accurate electricity price models are crucial tool for any decision-making by generating companies and costumers.

There is another important feature of electricity price formation – the instantaneous nature of the product. This results into that when the electricity is generated, due to the non-storability, it must be immediately delivered to the market (even if the electricity can be indirectly stored via hydroelectric schemes). That is why electricity production and

consumption must be continuously balanced. But if some shocks on supply or demand appear, and, they cannot be smoothed out, then this can have an important impact on electricity prices.

Electricity prices gather characteristics which reflect in the time series evolution and in which they differ from other commodities: high frequency, non constant mean, autocorrelation, non-normal distribution, heteroscedasticity, seasonality, high volatility and high frequency of occurrence of unusual prices etc. This can incur the occurrence of outages, blackouts, and price peaks, which happen seldom in a regulated environment. All those features incur high requirements placed on the empirical model.

Therefore, there is a wide range of papers and empirical studies concentrating on electricity price, its modeling and analyzing features of the time series of this commodity. Most of the authors pay attention to electricity price model development. Research is divided into development of statistical models (parameters of the price processes are estimated from the available historical market data) and fundamental models (price processes are modeled according to production costs and consumption). Results of statistical models confirm that the time series of the electricity prices have the tendency to revert to a long-term equilibrium level and are mostly modeled as mean-reversion jump diffusion processes, see Kian and Keyhani (2001), Escribano et al. (2002), Audet et al (2004), Deng and Jiang (2005), De Sanctis and Mari (2006), Kanamura and Ohashi (2007) etc. In contrast, some authors, see Rambhara et al. (2004), apply TAR model and conclude that results outperform traditional existing stochastic jump diffusion models for some regions if other effects (transitory spikes, temperature effects, etc.) are taken into account. Next, an occurrence of typical features in the time series has been deeply studied by a group of authors, see Shapiro and Wilk (1965), Box and Pierce (1970), Jarque and Bera (1980), Jarque and Bera (1981), Deng (1999), Barlow (2002), Geman and Roncoroni (2003), Fan and Yao (2005) etc. They concluded that some of the above mentioned features (jumps, high volatility, unusual prices, etc.) are the results of high sensitivity on changes in demand, shortages in electricity generation, forced outages, peaks in electricity demand, etc.

Thus in this paper, we propose the empirical model for modeling daily electricity prices in the three selected regions (California, North Europe and Austria), during the years 2006 – 2008; further, we provide the evidence that the non-linear models give better results than linear ones.

We estimate the empirical SETAR model and perform the full statistical verification of all estimated

parameters, and the models themselves, compare the results with each other and with the AR models. We place emphasis on the proof whether application of such non-linear models really give better results than linear ones. On the one hand, we prove that non-linear models really fit the data better than linear models, but, on the other hand, we show that employing non-linear SETAR models does not require the improvement of the diagnostic checks of residuals in regards to heteroscedasticity and non-normality testing.

The paper is organized as follows: first, general AR and SETAR model is described in Section 2 including the description of the methods for parameters estimation and confidence interval constructing. Section 3 is devoted to the statistical verification of the proposed model (testing of parameters significance and residuals checks). Results of the non-linear and linear model are compared in Section 4. Section 5 concludes this paper.

## 2. Model description

Let  $p$  is an integer positive number representing the order of auto-regressive model and let the time series  $y_t$  follow the AR model in the form of

$$y_t = \alpha_0 + \alpha_1 y_{t-1} + \dots + \alpha_p y_{t-p} + \varepsilon_t, \quad (1)$$

where  $\alpha$  are the slope parameters and  $\varepsilon_t$  is the error term.

When a process comprising more particular AR processes is considered, we can formulate a  $k$ -regime SETAR model that is followed by the time series  $y_t$  with threshold variable  $y_{t-d}$ , thus

$$y_t = \sum_{i=1}^k (\alpha_{i0} + \alpha_{i1} y_{t-1} + \dots + \alpha_{i, p_i} y_{t-p_i}) I(\cdot) + \varepsilon_t, \quad (2)$$

where  $p_1, \dots, p_k$  are the integer positive numbers representing the order of particular autoregressive models,  $d$  is a delay parameter,  $\alpha_{ij}$  are the slope parameters of  $i$ -th regime,  $I(\cdot)$  is the indicator function,

$$I(\cdot) = \begin{cases} 1 & \text{if } y_{t-d} \in C_i \\ 0 & \text{otherwise,} \end{cases} \quad (3)$$

and  $\varepsilon_t$  is the error term and  $\{C_i\}$  forms a partition of  $(-\infty, +\infty)$  in the sense that  $\sum_{i=1}^k C_i = (-\infty, +\infty)$  and  $C_i \cap C_j = \emptyset$  for all  $i \neq j$ .

A general two-regime SETAR model is defined as

$$y_t = \begin{pmatrix} \alpha_{10} + \alpha_{11}y_{t-1} + \dots \\ + \alpha_{1,p_1}y_{t-p_1} \end{pmatrix} I(y_{t-d} \leq r) + \begin{pmatrix} \alpha_{20} + \alpha_{21}y_{t-1} + \dots \\ + \alpha_{2,p_2}y_{t-p_2} \end{pmatrix} I(y_{t-d} > r) + \varepsilon_t, \quad (4)$$

where  $r$  is the threshold parameter. If the model takes on the form (4), the particular linear autoregressive process is partitioned by threshold value  $y_{t-d}$  with delay  $d$  and in assistance with threshold parameters  $r$ . In this paper, the process  $\{\varepsilon_t\}$  is assumed to be to be iid  $(0, \sigma^2)$ , although it can be heteroscedastic, as well.

## 2.1 Estimation of SETAR model

The model form of (4) can be rewritten in the following representation,

$$y_t = \mathbf{a}_1' \mathbf{Y}_{t,p_1} (I(y_{t-d} \leq r)) + \mathbf{a}_2' \mathbf{Y}_{t,p_2} (I(y_{t-d} > r)) + \varepsilon_t, \quad (5)$$

where  $\mathbf{Y}_{t,p_i} = (1, y_{t-1}, \dots, y_{t-p_i})'$  and  $\mathbf{a}_i = (\alpha_{0,i}, \alpha_{1,i}, \dots, \alpha_{p_i,i})'$  for  $i = 1, 2$ .

The unknown parameters  $\mathbf{a} = (\mathbf{a}_1', \mathbf{a}_2')'$  and the threshold parameter  $r$  must be estimated from the observed data  $\mathbf{Y} = (y_1, \dots, y_T)$ , moreover delay parameter  $d$  the order of  $p_i$  is also necessary to be determined. For this purpose, the sequential conditional LS estimator is employed under the auxiliary condition that process  $\varepsilon_t$  is iid  $(0, \sigma^2)$ . Under this condition, the LS estimator is equivalent to the maximum likelihood estimation.

LS estimation of parameters for a given value of  $r$  is as follows,

$$\hat{\mathbf{a}}(r) = \left( \sum_{t=1}^T \mathbf{Y}_t(r) \mathbf{Y}_t(r)' \right)^{-1} \left( \sum_{t=1}^T \mathbf{Y}_t(r) y_t \right), \quad (6)$$

with residuals  $\hat{\varepsilon}_t(r) = y_t - \hat{\mathbf{a}}(r)' \mathbf{Y}_t(r)$  and residual variance

$$\hat{\sigma}_e^2(r) = \frac{1}{T} \sum_{t=1}^T \hat{\varepsilon}_t(r)^2. \quad (7)$$

In order to estimate the parameter  $r$ , ordinary LS regression is run, setting  $r = y_{t-1}$  for all  $y_{t-1} \in R$  and for each regression, the residual variance  $\hat{\sigma}_e^2(r)$  is computed, then founded the value of  $r$  corresponding to the smallest variance, thus

$$\hat{r} = \min_{r \in R} \hat{\sigma}_e^2(r), \quad (8)$$

where  $R = [\underline{r}, \bar{r}]$  is a set of all possible threshold parameters comprising all observed data and  $\underline{r} = \min(\mathbf{Y})$ ,  $\bar{r} = \max(\mathbf{Y})$ . It is obvious that one needs to run  $T$  regressions in order to find parameters  $\hat{\mathbf{a}} = \hat{\mathbf{a}}(\hat{r})$ .

The same problem arises in the determination of the delay variable  $d \in [1, \bar{d}]$ , where  $\bar{d}$  is the maximum considered delay. It follows, that the number of the  $T$  regressions is not final. The minimization problem of (8) is augmented to include a search over  $d$ , so instead of  $T$  regressions, the search method requires  $T\bar{d}$  number of regressions, these parameters are used for the slope parameters estimation satisfying following function,

$$(\hat{r}, \hat{d}) = \min_{r \in R, d} \hat{\sigma}_e^2(r, d). \quad (9)$$

Finally, we add some remarks concerning the practical implementation of this framework. When employing non-linear models, practitioners can find several appropriate models for fitting data. Therefore, there also exists some goodness of fit measures of an estimated model. We present Akaike's information criterion for  $k$  regimes in the form of

$$AIC = \sum_{i=1}^k T_i \ln \hat{\sigma}_i^2 + 2(p_i + 1). \quad (10)$$

Next, it is also necessary to note that for the purpose of reliable model reliability, a set of threshold parameters  $R$  must be selected so that each regime contains the sufficient observations. Therefore, the set of threshold parameter  $R$  is not bounded by the observed data, but by the technique ensuring a sufficient number of observations in each regime. For instance, the 15<sup>th</sup> and 85<sup>th</sup> quartile are used to determinate the boundary of set  $R$ .

## 2.2 Confidence intervals

To test the statistical significance of estimated parameters, firstly, the confidence intervals must be constructed. First, we explain the difficulties occurring in the confidence interval construction of threshold parameters and afterwards the same problems for the slope parameters will be presented.

The confidence interval of threshold parameter is given by

$$\hat{\Gamma}(r) = \{r : LR_T(r) \leq z_\beta\}, \quad (11)$$

where  $LR_T$  is the likelihood ratio for the null hypothesis  $H_0 : \hat{r} = r_0$  in the form of

$$LR_T(r) = T \left( \frac{\hat{\sigma}_\varepsilon^2(r_0) - \hat{\sigma}_\varepsilon^2(\hat{r})}{\hat{\sigma}_\varepsilon^2(\hat{r})} \right) \quad (12)$$

and  $z_\alpha$  is  $\alpha$ -level critical value that is available in Hansen (1997, 2000). The graphical method of finding the confidence interval relies on plotting values of  $LR_T(r)$  against  $r$  and drawing the horizontal line at value of  $z_\alpha$ . Needless to say, there might arise a problem in practice because the region can be disjointed and, for that reason, applicably difficult. Therefore, the convexified region is constructed and used  $\hat{\Gamma}(r) = [\underline{r}, \bar{r}]$ , where  $\underline{r} = \{r : \min(\hat{\Gamma}(r))\}$  and  $\bar{r} = \{r : \max(\hat{\Gamma}(r))\}$ .

The confidence interval of the estimated slope parameters can be constructed in the standard way as in the case of linear models. Let  $\hat{r}$  be an estimated threshold parameter, the  $\beta$ -level confidence interval  $\hat{\Theta}_\alpha(\hat{r})$  of the slope parameters  $\hat{\alpha}$  is given by

$$\hat{\Theta}_\alpha(\hat{r}) = \hat{\alpha}(\hat{r}) \pm z_\beta \hat{s}(\hat{r}), \quad (13)$$

where  $z_\beta$  is  $\beta$ -level critical value for the normal distribution and  $\hat{s}(\hat{r})$  denotes a standard error. Hansen (1996) pointed out, that such constructed confidence intervals are not reliable in the case of the finite sample because the threshold parameter does not have to be estimated very precisely and can contaminate the estimate of  $\hat{\alpha}$ . Therefore, he proposed to take the union of all constructed confidence interval of  $\hat{\alpha}(r)$  for all  $r \in \hat{\Gamma}(r)$ , thus

$$\hat{\Theta}_\alpha = \bigcup_{r \in \hat{\Gamma}(r)} \hat{\Theta}(r). \quad (14)$$

### 3. Model verification

After the model is estimated, it is necessary to verify it. First and foremost, the estimated model of (5) has to be statistically significant relative to the linear AR model. Consequently, obtained residuals  $\hat{\varepsilon}_t(r)$  have to meet the assumption of the white noise and the slope parameters are needed to be statistically significant.

Firstly, we show the linearity test according to Hansen (1996, 1997) under the conditions that the parameter  $r$  is known and  $\varepsilon_t$  is assumed to be iid. The relevant null hypothesis  $H_0 : \hat{\alpha}_1 = \hat{\alpha}_2$  is tested against alternative hypothesis  $H_1 : \hat{\alpha}_1 \neq \hat{\alpha}_2$ .

The relevant  $F$ -statistics  $F_T(\hat{r})$  is equivalent to the supremum over the set  $R$  of the point-wise test-statistic  $F_T(r)$ ,

$$F_T(\hat{r}) = \sup_{r \in R} F_T(r), \quad (15)$$

where

$$F_T(r) = T \left( \frac{\hat{\sigma}_\varepsilon^2 - \hat{\sigma}_\varepsilon^2(r)}{\hat{\sigma}_\varepsilon^2(r)} \right) \quad (16)$$

and

$$\hat{\sigma}_\varepsilon^2 = \frac{1}{T} \sum_{t=1}^T (Y_t - \mathbf{Y}_t' \tilde{\alpha}_{AR})^2, \quad (17)$$

$$\tilde{\alpha}_{AR} = \left( \sum_{t=1}^T \mathbf{Y}_t \mathbf{Y}_t' \right)^{-1} \left( \sum_{t=1}^T \mathbf{Y}_t Y_t \right) \quad (18)$$

is OLS estimate of the parameters  $\tilde{\alpha}_{AR}$  from linear autoregressive model of order  $p$ .

When testing the slope parameters, one is allowed to use standard  $T$ -test by employing the united confidence interval from (14), see Hansen (1996, 2000).

The last step of the statistical verification consists in the necessity to perform diagnostic checks of residuals  $\hat{\varepsilon}_t(r)$ . Some tests that are used in the traditional linear framework can be applied for testing of the non-linear models. For instance, Jarque-Bera test can be used for normality testing in both frameworks. On the other hand, common Ljung-Box test does not remain valid; see Eitrheim and Teräsvirta (1996).

Here, generalized LM test is employed for serial correlation in AR( $p$ ) model of Breusch and Pagan (1979), which is based on the auxiliary regression,

$$\begin{aligned} \hat{\varepsilon}_t = & \beta_1 y_{t-1} + \dots + \beta_p y_{t-p} \\ & + \delta_1 \hat{\varepsilon}_{t-1} + \dots + \delta_q \hat{\varepsilon}_{t-q} + v_t. \end{aligned} \quad (19)$$

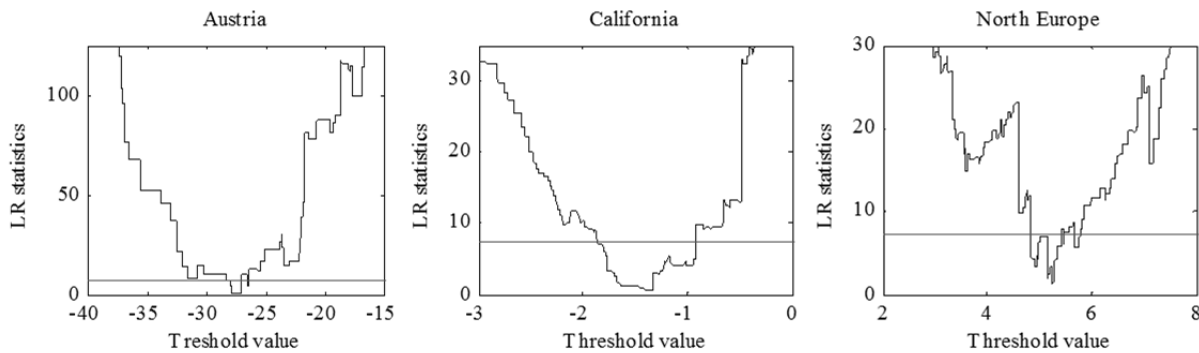
The LM test for  $q$ -th order serial dependence in  $\varepsilon_t$  is obtained as  $TR^2$ , where  $R^2$  is coefficient determination from the regression  $\hat{\varepsilon}_t$  on  $\hat{z}_t$ , where  $\hat{z}_t$  are relevant partial derivations of non-linear model, thus

$$\hat{\varepsilon}_t = \beta_1 \hat{z}_t + \dots + \beta_p \hat{z}_t + \delta_1 \hat{\varepsilon}_{t-1} + \dots + \delta_q \hat{\varepsilon}_{t-q} + v_t, \quad (20)$$

where

$$\hat{z}_t = \partial F(y_t; \hat{\theta}) / \partial \theta \quad (21)$$

and  $F(y_t; \hat{\theta})$  is non-linear SETAR model (4) and  $\hat{\theta} = (\hat{\alpha}_1, \hat{\alpha}_2, \hat{r}, \hat{d})$  are estimated parameters.



**Figure 1** Confidence intervals for threshold parameters

#### 4. Empirical results

In this part, the empirical results that we gathered in daily electricity price modeling by applying non-linear SETAR models will be presented. We employed this approach on daily electricity price data series from California (prices obtained from Energy Information Administration), North Europe (prices obtained from Nord Pool) and Austria (prices obtained from Energy Exchange Austria). Data sets obtained consists of eight annual time series from 2006 – 2008 and contains daily electricity prices. To obtain our data sample, we worked with discrete daily returns.

Firstly, we estimated all SETAR models and constructed confidence intervals of all parameters in accordance with Hansen (1997, 2000), but we perform the full enumeration in order to obtain the optimal order of particular AR regime and the optimal delay parameter. For them, we estimate the other model parameters. That is why we do not start with ACF and PACF functions as is usual in time series analysis.

Then we conducted the diagnostic checks and compared the gathered empirical results with linear models. Thus, we verify if the non-linear model really provides better results than the linear for the purpose of modeling time series. The accuracy of fitting the time series by non-linear and linear model was compared under criteria of residual variance and results of diagnostic checks were used. We also test whether the residuals variance of linear and non-linear models are statistically different.

##### 4.1 Model estimation

The crucial problem in SETAR estimation poses is a determination of delay parameter and order of the particular autoregressive process. The main slope parameters estimation follows immediately after that. One can explicitly define the delay parameter and order of the AR processes, but this approach is susceptible to be misspecified. Therefore, we employed a special algorithm representing the

complete enumeration of all possible models combining our conditions.

We form a set of possible delay parameter  $d \in \{1, 2, \dots, 5\}$ , a set representing the order of autoregressive model  $p_1, p_2 \in \{1, 2, \dots, 6\}$  and the set of threshold parameter  $r \in \{p_{0.15}, P_{0.85}\}$ , where  $p_{0.15}$  and  $P_{0.85}$  is 15<sup>th</sup> and 85<sup>th</sup> percentile. For all combinations, we estimated the slope parameters. We chosen order of autoregressive part corresponding to the minimal AIC criterion of (10) and selected the delay and threshold parameter corresponding to the minimal residual variance. Thus, we needed to perform 172,970 of estimations for each time series.

The estimation results are summarized in the following tables. Table 1 reports determined delay parameters, estimated threshold parameters and order of the autoregressive parts for each time series.

**Table 1** Threshold parameters and orders of delay and autoregressive parts

	North Europe	California	Austria
Delay parameter	1	1	1
Threshold parameter	5.1600	-1.3876	-27.0497

Next, the construction of confidence intervals follows. For this purpose, we perform a Monte-Carlo experiment. We generated 3,500 values of threshold parameters and for each of them we computed LR statistics according to (12). Then we plotted them against generated threshold parameters and draw the line  $z_\alpha$  at the critical value of 7.35, see Hansen (1997, 2000). Figure 1 depicts the results of our experiment.

We can see that the 95% confidence intervals are not really tight in some cases (especially for California) and are disjointed (Austria). Therefore, we have to convexify the obtained regions in accordance



with the above-mentioned technique in Section 2.2. Table 2 records the results.

**Table 2** Convexified 95% confidence intervals of threshold parameters

	North Europe	California	Austria
min	4.832	-1.850	-28.291
threshold parameter	5.160	-1.388	-27.050
max	5.788	-0.918	-26.366

For precision assessment of the estimated threshold parameters, we split our sample data into regime 1 for  $X_{t-d} < \underline{r}$  (left column in Table 3) and into regime 2 for  $X_{t-d} > \bar{r}$  (right column). We also extracted the data belonging to the gray zone, thus  $X_{t-d} \in \{\underline{r}, \bar{r}\}$ , see next Table.

**Table 3** Regime splitting of data

	Regime 1		Gray Zone		Regime 2	
North Europe	800	79.60%	36	3.58%	169	16.82%
California	403	40.10%	50	4.98%	552	54.93%
Austria	153	15.25%	5	0.50%	845	84.25%

The threshold parameters are estimated precisely in all cases, especially for Austria. For this time series, more than 99.5% of all observations fall into one of the two regimes with certainty and only 0.5 percent of all data are in gray zone.

Next Table 4 presents estimated slope parameters of particular regimes and for all time series. Statistically significant coefficients are in bold. In the last row of the Table, we show the number of observations used for slope parameters estimation in each regime.

**Table 4** Slope parameters of non-linear SETAR model

Parameter	North Europe		California		Austria	
	regime		regime		regime	
	1	2	1	2	1	2
p0	10.218	-3.310	5.091	-6.963	9.596	-44.809
p1	-0.515	0.005	0.255	-0.356	-0.237	-0.088
p2	-0.479	-0.008	-0.081	-0.363	-0.350	-0.286
p3	-0.372	-0.065	0.002	-0.239	-0.186	-0.225
p4	-0.498	-0.079	-0.040	-0.323	-0.168	-0.146
p5		-0.199	0.036	0.021	-0.097	-0.397
p6			-0.026	-0.062	0.017	
# of obs.	894	201	427	578	155	848

We can note here that particular AR processes of the SETAR model are not in higher order than 5 (all coefficients of the 6<sup>th</sup> order are not statistically

significant). Next, we can point out that while the model of North Europe and Austria comprise AR processes of incomplete order (some coefficients of lower order are not statistically significant), the orders of AR processes of the California SETAR model are complete.

The last step poses a diagnostic checking of residuals. This is presented in the next Section 4.2, where we compared non-linear models with linear autoregressive models. Nevertheless, we can say that estimated models face the same problem as many other models (heteroscedasticity and autocorrelation presence and non-normality of residuals). Our models are not an exception and they suffer from the same imperfections as the linear AR models, see next section.

## 4.2 Comparison with linear AR model

In this section, we provide a comparison of linear and non-linear AR models. Using the same data sets, we estimated linear AR model by employing LS estimator and determined the order process with a similar principle described in Section 4.1. Thus, for a particular value from a predetermined set of orders,  $p \in \{1, 2, \dots, 6\}$ , we run LS estimate and choose the one that is corresponding to the minimal residual variance. From obtained residuals, we also performed diagnostic checks and compared all results with non-linear SETAR models.

Next, Table 5 records the estimated slope parameters of AR models. Coefficients p1 – p6 are statistically significant.

**Table 5** Slope parameters of linear AR model

Parameter	North Europe	California	Austria
p0	0.025	0.626	0.173
p1	-0.326	-0.162	-0.530
p2	-0.367	-0.314	-0.644
p3	-0.301	-0.153	-0.545
p4	-0.328	-0.223	-0.566
p5	-0.347	-0.121	-0.596
p6	-0.233	-0.120	-0.486
# of obs.	1089	1089	1089

According to the results stated in Table 5, we can highlight that all parameters (except for constants) are statistically significant in contrast to parameters from non-linear models.

Hereafter, the main comparison of models is following. We perform the Levene test proposed by Levene (1960) which was found to be robust under non-normality and then F-test to test the non-linear

models against the linear. Next Table 6 summarizes all the results.

**Table 6** Variance summary

Model	North Europe	California	Austria
Variance of linear AR	100.317	150.304	450.888
Variance of non-linear SETAR	55.222	68.935	191.181
F statistics (p-values)	0.000	0.000	0.000
Levene test (p-values)	0.00000107	0.00000064	0.00000000

According to the results, it is obvious that the non-linear models are better and the residual variance of linear and non-linear models is statistically different. Further, we can see that the residual variances were decreased by nearly half.

Next, comparison consists in the potential improvement of the diagnostic checks under the conditions they give better results. Whereas we tested autocorrelation in linear models by Portmanteau test with order lag  $k=20$ , we are made to employ the modified Breusch-Pagan test (described in Section 3) in order to test the same lag order of autocorrelation in non-linear model. To detect heteroscedasticity, we employed the ARCH effect test and for testing of non-normality we used the Jarque-Bera test. Results in the form of  $p$ -values are summarized in Table 7.

It is apparent from the results, that using non-linear SETAR models does not improve the diagnostic check results significantly. The autocorrelation of residuals was getting rid of in the non-linear model of the California time series, but all the other results are almost the same. Heteroscedasticity is present for all time series and it is not dependent on the model employed. In every case, the ARCH effect test indicates the non-constant conditional residual variance. Thereafter, using the non-linear model is related to the following consequence: the normal distribution of residuals is more skewed in two cases (Nord Pool and Austria) and kurtosis is non-normal for both types of models. In the end, the Jarque Bera test of normality indicates non-normality of residuals for all estimated econometric models.

**Table 7** Diagnostic checks –  $p$ -values of particular tests

	Autocorrelation		Heteroscedasticity		Skewness		Kurtosis		Normality	
	lin	(non)	lin	(non)	lin	(non)	lin	(non)	lin	(non)
North Europe	0	0	0	0	0.006	0	0	0	0	0
California	0	0.674	0	0	0	0	0	0	0	0
Austria	0	0	0	0	0.417	0	0	0	0	0

## 5. Conclusion

In the paper, we proposed the empirical models for modeling daily electricity prices in the three selected regions (California, North Europe and Austria), during the period 2006 – 2008, performed the full statistical verification and further, we compared the models estimated with the empirical auto-regressive ones.

On the basis of the obtained results, we can conclude that non-linear models for electricity price modeling at all the selected markets fit the data better than the linear AR model. The residual variance of the non-linear models was half, in comparison to the residual variance of the linear model. On the other hand, using non-linear models did not improve the diagnostic checks and in our cases we obtained the same or very similar results. Next, we proved that while all slope parameters (except for constants) of AR models were significant, only some of the parameters of non-linear models were significant. Furthermore, the particular AR processes of SETAR models were of lower order than empirical AR models.

Generally, we can conclude that the non-linear SETAR models are more appropriate to model the electricity prices than the linear AR model, but neither of them captures the time-varying conditional variance and non-normality of probability distributions. This crucial problem lies in the fact that electricity prices are affected by many factors (i.e. seasonality, occurrence of price peaks, etc.) and the process of time series is characteristic with very high volatility and with high frequency of spikes resulting in the fact that electricity time series seems to be rather mean-reverting or even non-linearly mean-reverting. Furthermore, the dynamics of time series can be so tangled that the process comprises a lot of process.

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